REPORT DOCUMENTATION PAGE

AFRL-SR-BL-TR-01-

0427

1. REPORT DATE *(DD-MM-YYYY)*
21-06-2001

2. REPORT TYPE
Final Report

3. DATES COVERED *(From - To)*
Apr 1, 1998-Mar 30, 2001

4. TITLE AND SUBTITLE
Synthesis Methods for Distributed and Time Varying Controlled Systems

5a. CONTRACT NUMBER

5b. GRANT NUMBER
F49620-98-1-0416

5c. PROGRAM ELEMENT NUMBER

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

6. AUTHOR(S)
D'Andrea, Raffaello

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Cornell Universitv
Office of Sponsored Programs
120 Day Hall
Ithaca, NY 14853

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Marc Jacobs
AFOSR/NM
801 N. Randolph St. Room 7
Arlington, VA 22203-1977

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFOSR)
NOTICE OF TRANSMITTAL DTIC. THIS TECHNICAL REPORT HAS BEEN REVIEWED AND IS APPROVED FOR PUBLIC RELEASE LAW AFR 190-12. DISTRIBUTION IS UNLIMITED.

12. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
Prepared in cooperation with sub-contractor, Geir Dullerud, University of Illinois

14. ABSTRACT
The research entailed a computationally viable approach for analysis and control design of complex nonlinear systems. The developed techniques are directly applicable to a wide variety of engineering applications which are nonlinear and/or distributed, and are especially relevant to the analysis and control of complex vehicle systems. The accomplishment of these program goals was pursued from analytical, computation and experimental perspectives.

The main emphasis of this project was to develop not only a unifying framework but also new and relevant techniques for the control and analysis of systems that are time varying and distributed. The study of time varying systems was motivated by their direct application to the investigation of nonlinear systems along trajectories; this is a situation that arises frequently in general system analysis and simulation, as well as more specifically in control system design. The research proposed for distributed control was targeted at the growing number of engineering systems that are comprised of interconnected subsystems and must necessarily employ distributed control, as well as at the abundance of physical systems that are inherently distributed.

15. SUBJECT TERMS
Distributed Control, Robust Control, Spatially Distributed Systems

16. SECURITY CLASSIFICATION OF: U			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
					Raffaello D'Andrea
a. REPORT	b. ABSTRACT	c. THIS PAGE	UU	33	19b. TELEPHONE NUMBER *(include area code)* 607-255-0710

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39.18

20010810 101

SYNTHESIS METHODS FOR DISTRIBUTED AND TIME VARYING CONTROLLED SYSTEMS

F49620-98-1-0416

Raffaello D'Andrea
Sibley School of Mechanical and Aerospace Engineering
Cornell University
218 Upson Hall
Ithaca, NY 14850

Geir Dullerud
Mechanical and Industrial Engineering
University of Illinois, Urbana Champaign
MC-244, 1206 West Green Street
Urbana, IL 61801

Contents

1 Executive Summary

1.1 Accomplishments

In this program we developed a new computationally viable approach for analysis and control design of complex nonlinear systems. Resulting development techniques are directly applicable to a wide variety of engineering applications which are nonlinear and/or distributed, and are especially relevant to the analysis and control of complex vehicle systems. The accomplishment of these program goals was approached from analytical, computational and experimental perspectives.

The study of time varying systems was motivated by their direct application to the investigation of nonlinear systems along *trajectories*; this is a situation that arises frequently in general system analysis and simulation, as well as more specifically in control system design. The research in distributed control was targeted at the growing number of engineering systems that are comprised of interconnected subsystems and must necessarily employ distributed control, as well as at the abundance of physical systems that are inherently distributed. A general and computationally tractable theory was developed, and was verified using existing experimental facilities at Cornell and Illinois.

The research strategy for the program was leveraged from current robust control theory, which until now was limited to dealing with design and system analysis issues locally in the system envelope, and was not directly applicable in distributed scenarios. The research funded by this grant, however, coupled with the rapid advancements in the field of semi-definite programming (SDP), have formed the basis for a computational theory for control design based on SDP techniques.

The specific accomplishments were the following:

1. The generalization of the most important results in one dimensional system analysis and design to the multidimensional case. This was achieved by considering systems which can be expressed as linear fractional transformations on temporal and spatial shift operators. This formulation has resulted in tractable algorithms for controlling distributed systems, and has laid the foundation for extending these results to include system uncertainty, subject to a rich class of input disturbances.
2. The investigation of decentralized implementation of multidimensional controllers which result from multidimensional system optimization.
3. The generalization of the most important results in linear time invariant analysis to a linear time varying setting. One of the major directions was that of μ-theory for parametric, LTI and LTV perturbation classes which have been extensively studied by Doyle and co-workers, and of robust performance to uncertainty in the presence of white noise. A key component of this work was to obtain feasible computational schemes.

4. The development of optimal control strategies for linear time varying systems, with both H-infinity and H2 performance criteria. These schemes were combined with the aforementioned robust analysis results to provide general tools for robust, nonlinear controller synthesis along trajectories.
5. The investigation of model validation of nonlinear systems over large ranges of the state space using linear time varying techniques.
6. The exploration of model reduction of nonlinear systems along trajectories with guaranteed error bounds.
7. The systematic study of analysis and approximation of continuous linear time varying systems via discrete time varying systems.
8. Establishing connections between the results on linear time varying analysis and design to those on multidimensional optimization. One framework for control design and analysis was developed that includes these two problems as special cases. This framework permits control design and analysis for spatially varying distributed systems along trajectories, applications which arise, for example, from tight formation flight of unmanned aerial vehicles.

1.2 Personnel Supported

Raffaello D'Andrea Assistant Professor, Mechanical and Aerospace Engineering, Cornell University.

Geir Dullerud Assistant Professor, Mechanical and Industrial Engineering, University of Illinois at Urbana Champaign.

Christopher Gibson Graduate Research Assistant, Mechanical and Aerospace Engineering, Cornell University.

Aiguo Ying Graduate Research Assistant, Mechanical and Aerospace Engineering, Cornell University.

Jeffrey Fowler Graduate Research Assistant, Mechanical and Aerospace Engineering, Cornell University.

Matthew Earl Graduate Research Assistant, Theoretical and Applied Mechanics, Cornell University.

Andrew Eichelberger Master of Engineering Graduate Student, Mechanical and Aerospace Engineering, Cornell University.

James Leiz Master of Engineering Graduate Student, Mechanical and Aerospace Engineering, Cornell University.

Ryusei Matsumoto Master of Engineering Graduate Student, Mechanical and Aerospace Engineering, Cornell University.

Hoon Kong Master of Engineering Graduate Student, Mechanical and Aerospace Engineering, Cornell University.

Mazen H. Farhood Master of Science, Mechanical and Industrial Engineering, University of Illinois.

1.3 Publications

Refereed Journals

[1] R. D'Andrea. Linear matrix inequality conditions for robustness and control design. *International Journal of Robust and Nonlinear Control*, 2001. Accepted for publication.

[2] C.L. Pirie and G.E. Dullerud. Robust synthesis for uncertain time-varying systems. *SIAM Journal of Control and Optimization*, 2001. Accepted for publication.

[3] R. D'Andrea. Convex and finite dimensional conditions for controller synthesis with dynamic integral constraints. *IEEE Transactions on Automatic Control*, 46(2):222–234, 2001.

[4] R. D'Andrea. Extension of Parrott's theorem to non-definite scalings. *IEEE Transactions on Automatic Control*, 45(5):937–940, 2000.

[5] R. D'Andrea. Generalized ℓ_2 synthesis. *IEEE Transactions on Automatic Control*, 44(6):1145–1156, 1999.

Refereed Proceedings

[1] M. Farhood and G.E. Dullerud. Time-varying lpv systems. In *Proc. IEEE Conference on Decision and Control*, 2001. Accepted for publication.

[2] C. Pirie and G. E. Dullerud. Robust synthesis of uncertain time varying systems. In *Proc. American Control Conference*, pages 1842–1847, 2000.

[3] R. D'Andrea and C. Beck. Temporal discretization of spatially distributed systems. In *Proc. IEEE Conference on Decision and Control*, pages 197–202, 1999.

[4] R. D'Andrea. Convex and finite dimensional conditions for controller synthesis with dynamic integral constraints. In *Proc. IEEE Conference on Decision and Control*, pages 976–981, 1999.

[5] G. Dullerud and R. D'Andrea. Distributed control of inhomogeneous systems, with boundary conditions. In *Proc. IEEE Conference on Decision and Control*, pages 186–190, 1999.

[6] C. L. Beck and R. D'Andrea. Simplification of spatially distributed systems. In *Proc. IEEE Conference on Decision and Control*, pages 620–625, 1999.

[7] R. D'Andrea. Software for modeling, analysis, and control design for multidimensional systems. In *IEEE International Symposium on Computer-Aided Control System Design*, pages 24–27, 1999.

[8] R. D'Andrea. Linear matrix inequalities, multidimensional system optimization, and control of spatially distributed systems: An example. In *Proc. American Control Conference*, pages 2713–2717, 1999.

[9] R. D'Andrea. Generalized ℓ_2 synthesis with dynamic constraints. In *Mathematical Theory of Networks and Systems*, pages 117–120, 1998.

[10] R. D'Andrea, G. Dullerud, and S. Lall. Convex ℓ_2 synthesis for multidimensional systems. In *Proc. IEEE Conference on Decision and Control*, pages 1883–1888, 1998.

[11] G. E. Dullerud, R. D'Andrea, and S. Lall. Control of spatially varying distributed systems. In *Proc. IEEE Conference on Decision and Control*, pages 1889–1893, 1998.

[12] R. D'Andrea. A linear matrix inequality approach to decentralized control of distributed parameter systems. In *Proc. American Control Conference*, pages 1350–1354, 1998.

[13] C. L. Beck and R. D'Andrea. Computational study and comparisons of LFT reducibility methods. In *Proc. American Control Conference*, pages 1013–1017, 1998.

Submitted for Publication

[1] M. Farhood and G. E. Dullerud. Control of nonstationary LPV systems. *SIAM Journal of Control and Optimization*. Submitted for Publication.

[2] R. D'Andrea and G. E. Dullerud. Distributed control of spatially interconnected systems. *IEEE Transactions on Automatic Control*. Submitted for publication.

[3] R. D'Andrea. Temporal discretization of spatially interconnected systems. *Systems and Control Letters*. Submitted for publication.

[4] C. Beck and R. D'Andrea. Minimality, controllability, and observability for a class of multi-dimensional systems. *Automatica*. Submitted for Publication.

[5] G. E. Dullerud and R. D'Andrea. Distributed control of heterogeneous systems. *IEEE Transactions on Automatic Control*. Submitted for Publication.

Theses

[1] M. H. Farhood. Control of nonstationary LPV systems. Master's thesis, University of Illinois, 2001.

[2] C. M. Gibson. Application of a LMI optimal control technique for time varying systems to the experimental trajectory tracking of an autonomous gliding vehicle. Master's thesis, Cornell Univeristy, 2000.

[3] C. Pirie. Controller synthesis for uncertain time varying discrete time systems. Master's thesis, University of Waterloo, 1999.

Technical Reports

[1] S. Chang. Design and construction of an experimental platform for wing control. Technical report, Cornell University, 2001.

[2] H. Kong. An aircraft formation flight experiment. Technical report, Cornell University, 2000.

[3] R. Matsumoto. Design and construction of experimental platforms for the development of nonlinear system controllers. Technical report, Cornell University, 2000.

[4] J. Leiz. Experimental platform for control along trajectories. Technical report, Cornell University, 1998.

1.4 Interactions and Transitions

Conference Presentations

1. G. E. Dullerud. "Robust Synthesis of Uncertain Time Varying Systems." American Control Conference, Chicago, Illinois, June 2000.

2. R. D'Andrea. "Simplification of Spatially Distributed Systems." IEEE Conference on Decision and Control, Phoenix, Arizona, December 1999.

3. G. E. Dullerud. "Distributed Control of Inhomogeneous Systems, with Boundary Conditions." IEEE Conference on Decision and Control, Phoenix, Arizona, December 1999.

4. R. D'Andrea. "Temporal discretization of spatially distributed systems." IEEE Conference on Decision and Control, Phoenix, Arizona, December 1999.

5. R. D'Andrea. "Convex and finite dimensional conditions for controller synthesis with dynamic integral constraints." IEEE Conference on Decision and Control, Phoenix, Arizona, December 1999.

6. R. D'Andrea. "Software for modeling, analysis, and control design for multidimensional systems." IEEE International Symposium on Computer-Aided Control System Design, Kona, Hawaii, August 1999.

7. R. D'Andrea. "Linear matrix inequalities, multidimensional system optimization, and control of spatially distributed systems." American Control Conference, San Diego, California, June 1999.

8. R. D'Andrea. "Convex ℓ_2 synthesis for multidimensional systems." IEEE Conference on Decision and Control, Tampa, Florida, December 1998.

9. G. E. Dullerud. "Control of Spatially Varying Distributed Systems." IEEE Conference on Decision and Control, Tampa, Florida, December 1998.

10. R. D'Andrea. "Generalized ℓ_2 synthesis with dynamic constraints." Mathematical Theory of Networks and Systems Conference, Padova, Italy, July 1998.

11. R. D'Andrea. "A linear matrix inequality approach to decentralized control of distributed parameter systems." American Control Conference, Philadelphia, Pennsylvania, June 1998.

Invited Lectures and Seminars

1. G. E. Dullerud. Department of Cybernetics, Norwegian University of Science and Technology, Norway. August 2000. " Distributed and LTV Control of Heterogeneous Systems."

2. G. E. Dullerud. Department of Automatic Control, Lund Institute of Technology, Sweden. August 2000. " Distributed and LTV Control of Heterogeneous Systems."

3. G. E. Dullerud. Canadian Applied Mathematics Society Annual Meeting, Hamilton, Canada. June 2000. "Distributed Control of Heterogeneous Systems."

4. R. D'Andrea. Lund Institute of Technology, Department of Automatic Control, Lund, Sweden. June 2000. "Robust and Optimal Control of Complex Interconnected Systems."

5. R. D'Andrea. ETH Zurich, Automatic Control Laboratory, Zurich, Switzerland. June 2000. "Robust and Optimal Control of Complex Interconnected Systems."

6. R. D'Andrea. Lucent Technologies, Murray Hill, New Jersey. May 2000. "Robust and Optimal Control of Complex Interconnected Systems."

7. R. D'Andrea. University of California, San Diego, Mechanical and Aerospace Engineering. May 2000. "Robust and Optimal Control of Complex Interconnected Systems."
8. G. E. Dullerud. Iowa State University, Electrical and Computer Engineering, Ames, Iowa. November 1999. "Control Design of Complex Engineering Systems."
9. R. D'Andrea. California Institute of Technology, Control and Dynamical Systems, Pasadena, CA. November 1999. "Robust and Optimal Control of Complex Interconnected Systems."
10. R. D'Andrea. Princeton University, Mechanical and Aerospace Engineering, Princeton, NJ. October 1999. "Robust and Optimal Control of Complex Interconnected Systems."
11. R. D'Andrea. Yale University, Electrical Engineering, New Haven, CT. September 1999. "Robust and Optimal Control of Complex Interconnected Systems."
12. R. D'Andrea. Universal Instruments Corporation, Binghamton, NY. September 1999. "Feedback Control: Overcoming Complexity and Uncertainty."
13. R. D'Andrea. Wright Patterson Air Force Base, Air Vehicles Directorate, Dayton, OH. July 1999. "Decentralized Control of Spatially Distributed Systems, with Application to Formation Flight."
14. R. D'Andrea. University of Toronto, Canada, Electrical and Computer Engineering. April 1999. "Robust and Optimal Control of Complex Interconnected Systems."
15. R. D'Andrea. Colorado State University, Electrical and Computer Engineering, Fort Collins, CO. March 1999. "Decentralized Control of Spatially Distributed Systems, with Application to Formation Flight."
16. R. D'Andrea. University of California, Los Angeles, Mechanical and Aerospace Engineering. March 1999. "Decentralized Control of Spatially Distributed Systems, with Application to Formation Flight."
17. R. D'Andrea. University of California, Santa Barbara, Center for Control Engineering and Computation. March 1999. "Decentralized Control of Spatially Distributed Systems, with Application to Formation Flight."
18. R. D'Andrea. Massachusetts Institute of Technology, Laboratory for Information and Decision Systems, Boston, MA. February 1999. "Decentralized Control of Spatially Distributed Systems, with Application to Formation Flight."
19. R. D'Andrea. Xerox Distinguished Lecture Series in Control and Diagnostics, J. C. Wilson Center for Research and Technology, Xerox Corporation, Webster, NY. November 1998. "Feedback Control: Overcoming Complexity and Uncertainty."

20. R. D'Andrea. University of Michigan, Mechanical Engineering, Ann Arbor, MI. October 1998. "Robust and Optimal Control of Complex Interconnected Systems."

21. R. D'Andrea. Air Force Office of Scientific Research Dynamics and Control Workshop, California Institute of Technology, Pasadena, CA. May 1998. "Robust Control of Complex Systems."

1.5 Honors and Awards

D'Andrea

RoboCup World Champions, 2000

Project manager and faculty advisor for the world champion F180 League Cornell Autonomous Robotic Soccer team. Melbourne, Australia 2000.

National Science Foundation CAREER Award, 2000

National Academy of Engineering Frontiers in Engineering Symposium, 2000

Selected to participate in symposium bring together engineers ranging in age from 30 to 45 years who are performing leading-edge research and technical work.

J.P. and Mary Berger '50 Excellence in Teaching Award, 2000

College of Engineering award, awarded annually to twenty outstanding teachers at Cornell University.

D. G. Shepherd Teaching Prize, 1999

Awarded annually to the most outstanding teacher in the Sibley School of Mechanical and Aerospace Engineering, Cornell University.

RoboCup World Champions, 1999

Project manager and faculty advisor for the world champion F180 League Cornell Autonomous Robotic Soccer team. Stockholm, Sweden 1999.

Dullerud

National Science Foundation CAREER Award, 1999

2 Spatially Interconnected Systems

2.1 Summary

Many systems consist of similar units which directly interact with their nearest neighbors. Even when these units have tractable models and interact with their neighbors in a simple and predictable fashion, the resulting system often displays rich and complex behavior when viewed as a whole. There are many examples of such systems, including:

- Automobiles on a freeway: during periods of congestion, drivers are typically concerned with the position and velocity of the vehicle directly in front and directly behind them. Even though a driver's response in these situations may be predictable and easy to model, the overall behavior of the vehicles on the freeway is very complex, and is prone to many types of instabilities. This has led to research in automated highway systems to increase vehicle throughput and eliminate traffic instabilities [1].
- Formation flight of uninhabited aerial vehicles: in these applications, unmanned vehicles are flown in close formation in order to increase the effective aspect ratio of the vehicles and thus reduce drag (migrating birds employ a similar strategy, and the resulting V-formations can be explained in terms of drag reduction [2]). The identical vehicles are coupled to their nearest neighbors aerodynamically, and any control system being sought must take this coupling into account to ensure that disturbances are not amplified as they propagate through the system [3], [4].
- Satellite formation flight: there have been recent proposals for utilizing formations of satellites which create sparse apertures for remote sensing applications. These configurations have the same performance as an extremely large satellite, but at a fraction of the cost. The required performance is the method by which these systems interact; the satellites must maintain a fixed formation in the presence of gravitational perturbations, solar radiation, and atmospheric drag, using a minimum amount of fuel for formation keeping [5], [6], [7], [8].
- Cross directional control: in these applications, arrays of spatially distributed sensors and actuators are used to maintain uniformity of the manufactured paper in the direction perpendicular to the travel of the sheet. The effects of actuation in one spatial location are propagated to neighboring locations by the very material being controlled [9], [10], [11].
- Certain classes of partial differential equations: many PDEs are derived by considering the interaction of an infinite number of infinitesimal elements interacting with their nearest neighbors; examples include the deflection of beams, plates, and membranes, and the temperature distribution of thermally conductive materials [12].

An important aspect of many of these systems is that sensing and actuation capabilities exist at every unit. In the examples above, this is clearly the case for vehicle platoons, aerial vehicle systems, satellite constellations, and cross-directional control systems. With the rapid advances in micro electro-mechanical actuators and sensors, one may control the vibrations of plates by instrumenting them with a large number of distributed actuators and sensors as well. This latter application is typically referred to as smart structure control; an excellent treatment of this emerging field may be found in [13].

If one attempts to control these systems using standard control design techniques, severe limitations will quickly be encountered as most optimal control techniques cannot handle systems of very high dimension and with a large number of inputs and outputs. It is also not feasible to control these systems with centralized schemes, as these require high levels of connectivity, impose a substantial computational burden, and are typically more sensitive to failures and modeling errors than decentralized schemes.

In order for any optimal control technique to be successful, the structure of the system must be exploited in order to obtain tractable algorithms. In particular, a large class of problems can be captured with the following model equations:

$$\begin{bmatrix} w_{\mathrm{T}}(t,\mathrm{s}) \\ w_{\mathrm{S}}(t,\mathrm{s}) \\ z(t,\mathrm{s}) \end{bmatrix} = \begin{bmatrix} A_{\mathrm{TT}} & A_{\mathrm{TS}} & B_{\mathrm{T}} \\ A_{\mathrm{ST}} & A_{\mathrm{SS}} & B_{\mathrm{S}} \\ C_{\mathrm{T}} & C_{\mathrm{S}} & D \end{bmatrix} \begin{bmatrix} x_{\mathrm{T}}(t,\mathrm{s}) \\ x_{\mathrm{S}}(t,\mathrm{s}) \\ d(t,\mathrm{s}) \end{bmatrix}, \tag{1}$$

$$w_{\mathrm{T}}(t) = \dot{x}_{\mathrm{T}}(t), \tag{2}$$

$$w_{\mathrm{S}}(t) = \Delta_{\mathrm{S,m}} x_{\mathrm{S}}(t). \tag{3}$$

In the description above, the signals, such as $d(t,\mathrm{s})$, are indexed by a temporal independent variable t and spatial independent variables $\mathrm{s} = (s_1, s_2, \cdots, s_L)$. In other words, the system evolves in both time and space, where the number of spatial independent variables is L.

Define the shift operators S_i as follows

$$(\mathrm{S}_i u(t))(\mathrm{s}) := u(t, s_1, \cdots, s_i{+}1, \cdots, s_L), \quad i = 1, \cdots, L. \tag{4}$$

For a given $(2L{+}1)$-tuple of non-negative integers $\mathbf{m} = (m_0, m_1, m_{-1}, m_2, m_{-2}, \cdots, m_{-L})$, we define the following structured operator:

$$\Delta_{\mathrm{S,m}} := \mathbf{diag}(\mathrm{S}_1 I_{m_1}, \mathrm{S}_1^{-1} I_{m_{-1}}, \mathrm{S}_2 I_{m_2}, \mathrm{S}_2^{-1} I_{m_{-2}} \cdots, \mathrm{S}_L^{-1} I_{m_{-L}}). \tag{5}$$

Thus operator $\Delta_{\mathrm{S,m}}$ captures the spatial evolution of the system.

In the series of papers [14], [15], [16], and [17], control design and analysis for these system is considered, which fall under the class of spatially invariant systems. Other researchers

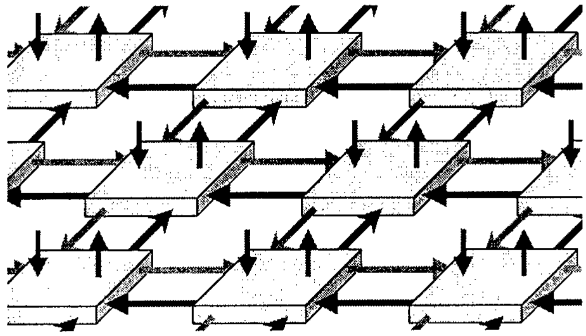

Figure 1: Controller implementation for two spatial dimensions. Each subsystem is a finite dimensional, linear time invariant subsystem which is connected to its nearest neighbors. The signals $y = y(t, s_1, s_2)$ coming into each subsystem are the local sensor variables, and the signals $u = u(t, s_1, s_2)$ leaving each subsystem are the actuator signals.

have proposed methods for designing controllers for spatially invariant systems by taking Fourier transforms in the spatial independent variables, and designing controllers as a function of the spatial frequency variables; see [18], [19], [20], and the references therein. The strengths of the techniques developed as part of the research funded under this grant are:

- No "gridding" of the spatial frequency transform variables is required; gridding the spatial frequency variables can quickly become computationally prohibitive, especially when dealing with more than one spatial dimension.
- The non-trivial problem of interpolating the spatial frequency-dependent controllers is avoided.
- The resulting controllers inherit the same structure as the plant, which implies that the resulting controllers have a simple implementation. This is depicted in Figure 1.
- The formalism for treating these classes of problems can be readily extended to the spatially and time varying case.
- The techniques are based on linear matrix inequalities (LMIs). LMIs appear to be a unifying thread in robust and optimal control design and analysis, increasing the likelihood of extending the existing, standard robust and optimal control design tools to spatially distributed systems.

2.2 Software

A MATLAB toolbox was developed in order to facilitate control design and analysis for these classes of systems, the Multidimensional Systems toolbox. This software allows one to perform the following tasks:

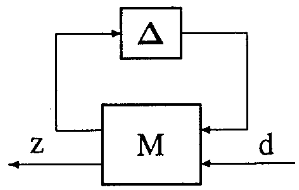

Figure 2: Uncertainty and Linear Fractional Representations

1. Construct MD system representations from first principles models;
2. Model reduce MD systems;
3. Manipulate MD systems, such as addition, multiplication, block transformations, etc.;
4. Design spatially distributed controllers for MD systems;
5. Analyze the performance of MD systems;
6. Simulate MD systems

2.2.1 Data Structure

A MD system **sys** is captured by the following structured object:

sys.A	n by n real matrix
sys.B	n by m real matrix
sys.C	p by n real matrix
sys.D	p by m real matrix
sys.blk	L by 1 real array

Define $\boldsymbol{\Delta} = \mathbf{diag}(\delta_1 I_{\text{sys}.blk(1)}, \cdots, \delta_L I_{\text{sys}.blk(L)})$, where the $\boldsymbol{\delta}_i$ are scalar indeterminates, and **sys**.blk captures the multiplicity of each indeterminate. **sys** captures the following structured object:

$$\text{sys}.D + \text{sys}.C\boldsymbol{\Delta}(I - \text{sys}.A\boldsymbol{\Delta})^{-1}\text{sys}.B \tag{6}$$

Defining

$$M = \begin{bmatrix} \text{sys}.A & \text{sys}.B \\ \text{sys}.C & \text{sys}.D \end{bmatrix} \tag{7}$$

it corresponds to the object in Figure 2. This block diagram is often represented by the star product of Δ and M, $\Delta \star M$.

The functions in the MD toolbox can be divided into six parts. We outline the various functions in the MD toolbox by considering a simple example: the 2D heat equation:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + Q \tag{8}$$

where $U(x, y, t)$ is the temperature of the plate, Q is the heat added to the plate, x and y are the independent spatial coordinates, and t is time.

Define the following operators:

$$\mathbf{X} := \frac{\partial}{\partial x}, \quad \mathbf{Y} := \frac{\partial}{\partial y}, \quad \mathbf{T} := \frac{\partial}{\partial t} \tag{9}$$

Then the 2D heat equation can be captured as follows:

$$\mathbf{T}U = (\mathbf{X}^2 + \mathbf{Y}^2)U + Q \tag{10}$$

2.2.2 Modeling of MD systems

The function **MN2ONMD**, which corresponds to *Multinomial to Output Nulling*, creates an output nulling [21] representation of the form

$$(\Delta \star M_{ON})w = 0. \tag{11}$$

The w capture all the variables in the multidimensional system; for our example, $w = (U, Q)$. **MN2ONMD** takes as an input a two dimensional cell array R and returns **sys**, where **sys** is used to capture Δ and M. For our example, Δ is of the form $\mathbf{diag}(\mathbf{T}I, \mathbf{X}I, \mathbf{Y}I)$: R is used to capture the multinomial which describes our system as a set of multinomial equations; it is often the most natural way to describe a system from first principles [22], [23].

An input-output representation is created from an output nulling representation with function **ON2IOMD**; for example, one could set Q an input and U an output and construct the following representation:

$$U = (\Delta \star M_{IO})Q \tag{12}$$

The above functions can readily handle systems with many inputs and outputs, many equations, and many operators.

2.2.3 Model Reduction

The systems created via **MN2ONMD** tend to be very nonminimal. Functions **CTRBFMD** (decomposition of MD system into reachable and unreachable parts, see [24]), and **MINREALMD** (minimal realization by truncation of unreachable, unobservable parts) can be used to create minimal realizations when the indeterminates in $\mathbf{\Delta}$ do not commute. It is found in practice [25] that these algorithms yield minimal realizations even when the indeterminates in $\mathbf{\Delta}$ commute.

2.2.4 System Manipulation

These functions allow one to interconnect, add, subtract, concatenate, etc. MD systems. For example, one may add the following equation to the heat equation system

$$y = U + n \tag{13}$$

by first converting the above to an ON system via **MN2ONMD**, and then using **AUGMD** (which corresponds to augment) to incorporate this additional system of equations to the 2D heat equation. y in the above equation could correspond to the sensed variable, and n the sensor noise.

Other functions are used to manipulate the blocks in $\mathbf{\Delta}$. These manipulations include bilinear transformations, continuous to discrete time sampling, reordering of blocks, and the inversion of blocks. For example, in the 2D heat equation example, one may want to invert the indeterminate corresponding to $\mathbf{T}$ and express the system using indeterminate $\mathbf{T}^{-1}$ using function **LFTINVMD**. This would be done to cast the system in standard state space form (note that a 1D state space system can be thought of as an LFT between $\mathbf{T}^{-1}$ and a constant matrix).

2.2.5 Controller Synthesis

These functions allow one to design controller for MD systems. For example, for the 2D heat equation example, one may want to design a MD system with input y and output Q which stabilizes the heat equation and which rejects sensor noise n. The reader is referred to [16], [17], and [26] for a description of these MD synthesis techniques.

2.2.6 System Analysis

These set of functions are used to analyze the $\mathcal{L}_2$ performance of MD systems.

2.2.7 Simulation

These set of functions allow one to simulate MD systems; this is particularly useful when implementing MD controllers designed using the MD controller synthesis techniques in the toolbox.

2.2.8 Complete List of Functions

The complete list of functions which currently constitute the MD Toolbox may be found in Table 1.

System Description	
MN2ONMD	- Construct an output nulling representation
ON2IOMD	- Output nulling to input-output
Model Reduction	
CTRBFMD	- Decomposition into reachable and unreachable parts
MINREALMD	- Minimal realization
System Manipulation	
ADDMD	- Add two systems
AUGMD	- Concatenate two systems
MULTMD	- Multiply two systems
STARMD	- Star product of two systems
SUBMD	- Subtract one system from another
Block Transformations	
BLMD	- General bilinear transformation
C2DMD	- Bilinear transformation, plane to disk
C2DZOHMD	- Zero order hold, continuous to discrete transformation
D2CMD	- Bilinear transformation, disk to plane
LFTINVMD	- Inverts indeterminates in system description
PERMUTEMD	- Combines two block structures into one
SHIFTMD	- Bilinear transformation, plane to plane
SWAPMD	- Swap blocks in realization
Controller Synthesis	
CHFMD	- Cts. time L2 to L2 control design
CHFOPTMD	- Optimal cts. time L2 to L2 control design
CNSTMD	- Explicit construction of controller
System Analysis	
CAUGMD	- Augments system to reduce conservatism of analysis LMIs
CLYAPMD	- Continuous time Lyapunov Analysis for stability
STABMD	- Stability test by frequency search
System Simulation	
CRCRSMD	- Decomposition to allow Taylor expansion
DSIMULATEMD	- Discrete time simulation

Table 1: Functions in MD Toolbox

3 Time and Spatially Varying Systems

3.1 Control of nonstationary systems

A substantial number of engineering control and analysis problems can be reduced to analysis along a nominal trajectory. A few examples are described below.

- An array of automatically controlled aircraft are required to follow a repeating hold pattern. The necessary commands to achieve this are computed, and an outer feedback control loop is used to ensure that despite the presence of wind and other disturbances the trajectories are closely followed.
- The load profile of a power system exhibits a repetitive fluctuation on a weekly (sometimes daily) cycle, and is thus periodic. A feedback control loop is used to stabilize the system around the desired power trajectory.
- A satellite is required to maintain a complex low-altitude orbit, but for energy conservation may only use its thrusters for short intervals at specified points in time. A feedback control law is used to ensure the correct trajectory above the earth. In this example the system is both intrinsically time varying and nonlinear.

In each example the systems follow prespecified trajectories, and the dynamics can be expected to be significantly nonlinear. Furthermore very accurate open-loop models for these systems would be extremely complex; consider simply the wing-fluid interaction in one aircraft. The PIs have pursued design and analysis methods for control of complex systems along prespecified trajectories using tools from control, dynamical systems and nonlinear systems theory in a hybrid combination. The particular approach described here concentrates on novel techniques developed by the PIs involving linear time varying systems.

Linear time varying (LTV) systems form an important and central class of systems for control design because they can be used to represent and analyze nonlinear systems along *trajectories*. The main advantage of considering LTV systems is analytical and computational, as they are considerably simpler than starting with general nonlinear systems. To make the connection between nonlinear systems along trajectories and LTV systems more concrete, linearization of trajectories is now reviewed. A discrete[1] nonlinear system can be described by

$$\begin{aligned} \bar{x}_{k+1} &= F(\bar{x}_k, \bar{w}_k) \\ \bar{z}_k &= H(\bar{x}_k, \bar{w}_k), \end{aligned} \tag{14}$$

[1]for clarity of exposition we focus on discrete systems

where $\bar{x}_k$ is the system state, $\bar{w}_k$ is an exogenous input, and $\bar{z}_k$ is the output. Suppose it is desired to analyze or maintain this system near the trajectory $\tilde{x}_k$ resulting from input $\tilde{w}_k$, giving output $\tilde{z}_k$. Then the linearization of this system along the trajectory is

$$\begin{aligned} x_{k+1} &= A_k x_k + B_k w_k \\ z_k &= C_k x_k + D_k w_k, \end{aligned} \tag{15}$$

where $[A_k \; B_k] = \frac{\partial F}{\partial q}|_{(\tilde{x}_k, \tilde{w}_k)}$ and $[C_k \; D_k] = \frac{\partial H}{\partial q}|_{(\tilde{x}_k, \tilde{w}_k)}$. By studying this LTV system, if the values of w_k are not too large, linearization theorems guarantee that the actual response of the nonlinear system to the input $\bar{w}_k = w_k + \tilde{w}_k$ will be near $\bar{z}_k = \tilde{z}_k + z_k$. Methods for predicting this mismatch and controlling the system around such trajectories is a main motivation of the LTV program pursued.

The approach described in this section is a synopsis of that given in earlier work by Dullerud and Lall, where the starting point for this formalism is an LTV system of the form in (15). The major feature of the framework now described is that it allows LTV systems to be treated almost as though they were LTI; matrices are replaced by block diagonal operators and transfer functions by a newly defined system function. The framework itself is very straightforward to describe and has substantial implications for the treatment of LTV systems, both from an analytical and a computational perspective. Indeed the examples shown on analysis and optimal synthesis clearly demonstrate the strength of this new framework as very complex manipulations appear transparent.

To illustrate the main idea consider the following important question that one might ask: what is the induced norm of the system on the finite energy signals ℓ_2? As is well known from the literature on uncertain systems, this question is fundamental to issues both for robustness analysis and for local stabilization of nonlinear systems.

In order to answer such questions, we rewrite the system described by (15) as

$$x = ZAx + ZBw \tag{16}$$

$$z = Cx + Dw. \tag{17}$$

Here $x = (x_0, x_1, x_2, \cdots)$, and similarly for u and y. The symbols A, B, C, and D represent *block diagonal* operators which have matrix representations, e.g.

$$\begin{bmatrix} A_0 & & & 0 \\ & A_1 & & \\ & & A_2 & \\ 0 & & & \ddots \end{bmatrix},$$

and the operator Z is the unilateral shift or delay operator on ℓ_2. Then the standard notion of exponential stability corresponds to invertibility of the operator $I - ZA$. If we write the map from w to z as G, then

$$G = C(I - ZA)^{-1}ZB + D$$

A key result is that this representation of time-varying systems allows the definition of a *system function* which formally plays the analogous role of a transfer function, and is defined by

$$\widehat{G}(\lambda) = C(I - \lambda ZA)^{-1}\lambda ZB + D. \tag{18}$$

Note that this function, although appearing nearly identical to a transfer function, is not a transfer function even when the given system is time invariant. A central result is the following induced norm equality

$$\|G\| = \max_{\lambda \in \bar{\mathbb{D}}} \left\| C(I - \lambda ZA)^{-1}\lambda ZB + D \right\|,$$

here $\mathbb{D}$ is the set of complex numbers of magnitude less than one. This can be shown using a combination of spectral theory for operators and complex analysis. A generalized version of the result can also be demonstrated when the system function is a function of several (or countable) variables; this multidimensional version is very important for work with model reduction and has a many significant connections with control methods. The main advantage of this frequency domain formulation is that it allows the techniques of robust analysis and synthesis to be applied directly to the LTV system. For example a structured version of the KYP lemma is

The norm inequality $\|G\| < 1$ is satisfied if and only if there exists a block diagonal *operator $X > 0$ such that the inequality*

$$\begin{bmatrix} ZA & ZB \\ C & D \end{bmatrix}^* \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} ZA & ZB \\ C & D \end{bmatrix} - \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} < 0 \quad \textit{is satisfied.} \tag{19}$$

This is a convex condition and reduces to a finite dimensional linear matrix inequality (LMI) when the time horizon is finite or the system G is periodic.

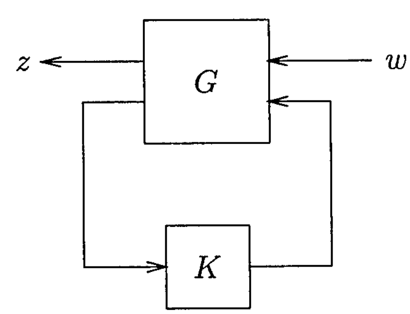

Figure 3: Closed-loop system

To further illustrate the perspective and power gained by applying this machinery a synthesis problem is now described. Let G be defined as above but described by the following

partitioned state space equations

$$
\begin{aligned}
x_{k+1} &= A_k x_k + B_{1k} w_k + B_{2k} u_k \qquad x_0 = 0 \\
z_k &= C_{1k} x_k + D_{11k} w_k + D_{12k} u_k \\
y_k &= C_{2k} x_k + D_{21k} w_k
\end{aligned} \tag{20}
$$

where $x_k \in \mathbb{R}^n$, $w_k \in \mathbb{R}^{n_w}$, $u_k \in \mathbb{R}^{n_u}$, $z_k \in \mathbb{R}^{n_z}$, and $y_k \in \mathbb{R}^{n_y}$. Make the physical and technical assumption that the matrices A, B, C, D are uniformly bounded functions of time.

Suppose this system is to be controlled by a controller K characterized by

$$
\begin{aligned}
x_{k+1}^K &= A_k^K x_k^K + B_k^K y_k \\
u_k &= C_k^K x_k^K + D_k^K y_k.
\end{aligned} \tag{21}
$$

where $x_k^K \in \mathbb{R}^m$. The connection of G and K is shown in Figure 3.

The synthesis question that corresponds to the analysis question above is: does there exist a controller K that stabilizes the closed-loop system and makes the map $\|w \mapsto z\| < 1$? A controller satisfying these conditions is said to be *admissible*; here by scaling it is again assumed, without loss of generality, that the norm bound sought is one.

By using the system function of (18), it is possible to derive the following synthesis theorem

There exists an admissible synthesis K for G, with state dimension no less than that of G, if and only if there exist block-diagonal operators $R > 0$ and $S > 0$ satisfying

(i) $$\begin{bmatrix} N_R & 0 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} ARA^* - Z^*RZ & ARC_1^* & B_1 \\ C_1RA^* & C_1RC_1^* - I & D_{11} \\ B_1^* & D_{11}^* & -I \end{bmatrix} \begin{bmatrix} N_R & 0 \\ 0 & I \end{bmatrix} < 0$$

(ii) $$\begin{bmatrix} N_S & 0 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} A^*Z^*SZA - S & A^*Z^*SZB_1 & C_1 \\ B_1^*Z^*SZA & B_1^*Z^*SZB_1 - I & D_{11}^* \\ C_1 & D_{11} & -I \end{bmatrix} \begin{bmatrix} N_S & 0 \\ 0 & I \end{bmatrix} < 0$$

(iii) $$\begin{bmatrix} R & I \\ I & S \end{bmatrix} \geq 0$$

where the operators N_R, N_S satisfy

$$
\begin{aligned}
\mathrm{Im} N_R &= \mathrm{Ker}\begin{bmatrix} B_2^* & D_{12}^* \end{bmatrix}, \; N_R^* N_R = I \\
\mathrm{Im} N_S &= \mathrm{Ker}\begin{bmatrix} C_2 & D_{21} \end{bmatrix}, \; N_S^* N_S = I.
\end{aligned}
$$

Note that these conditions look formally identical to those obtained in the optimal H_∞ synthesis results via LMIs in the stationary case. If the above conditions are met an explicit

controller can be determined from R and S, just as in the standard LMI solution to the H_∞ synthesis problem. The above general synthesis result is infinite dimensional and convex, but reduces to a finite dimensional convex condition for finite time horizons or periodic systems; virtually all scenarios of engineering interest fall into these categories.

During the project we have considerably extended the nonstationary problems that can be solved in the area of robust control, thus making new methods available for control along trajectories:

- We have developed new convex synthesis conditions for robust performance in linear time-varying systems, subject to time-varying perturbations. In particular these results are *exact* for sensitivity minimization in the presence of multiplicative perturbations. The methods apply to a number of additional robust performance problems, and are always both necessary and sufficient when the system plant is periodic. Otherwise the conditions provided are *always* sufficient, and a controller can be directly constructed when this condition holds.
- Considered control of nonstationary linear parameter-varying systems. This work has resulted in synthesis conditions derived for such systems using an operator theoretic framework with the ℓ_2 induced norm as the performance measure. These conditions are given in terms of structured operator inequalities. In general, evaluating the validity of these conditions is an infinite dimensional convex optimization problem; however, in certain important cases these reduce to finite dimensional semi-definite programming problem, and are thus readily verified.
- Have introduced and studied *eventually periodic* systems in the context of nonlinear systems which transition between equilibria or periodic orbits. It has been shown that general nonstationary analysis conditions are computable for such systems, and we have studied potential conservatism for synthesis problems as well.

3.2 Distributed control of heterogeneous systems

As detailed above, recently there has been renewed research interest in distributed control of systems with an emphasis on synthesizing controllers that preserve the distributed structure of the nominal plant. The primary motivation for this work is (1) the increasing number of systems that are formed by the interconnection of interacting subsystems; and (2) the tremendous potential of emerging technologies which are making the deployment of large distributed sensor and actuator arrays possible. Much of this recent work has focused on systems that are shift invariant, or *homogeneous*, with respect to both spatial and temporal variables. Many systems occurring in both nature and engineering, due to boundary conditions, inhomogeneity in material, or the coupling of differing subsystems, fail to possess such an invariance property, particularly with respect to spatial variables.

An achievement of the program has been the development of tools and results for distributed control, within the context of robust control, to deal with such *heterogeneity*. The PIs have considered distributed state space systems using a generalization of the Roesser model, and develop stability and synthesis results using the ℓ_2-induced norm as the performance measure. In particular the work is concerned with developing controllers which have the same state space structure as the nominal system. Our approach makes significant use of operator theory, and the results obtained are all stated as convex feasibility problems. More specifically, we extend robust control machinery to include heterogeneous Roesser systems. We derive sufficient conditions for analyzing performance with respect to the ℓ_2-induced norm, and then provide sufficient conditions for the existence controllers which stabilize the system and provide a guaranteed level of performance. The techniques developed are based on extending and combining those developed by the PIs on LTV systems and homogeneous distributed systems. An operator inertia concept is central to obtaining the results, and is adapted for use with distributed systems from earlier work on standard nonstationary systems. The analysis and synthesis results we derive are stated in terms of linear operator inequalities over infinite dimensional spaces. In certain circumstances they can be reduced directly to linear matrix inequalities, and thus their feasibility can be readily determined using semidefinite programming.

4 Formation Flight Test-Bed

4.1 Summary

An integral part of our research program was to evaluate the analytical tools being developed on several application systems and experiments. The main goals were to test the tools on real applications, to contribute to the understanding of several control applications and provide useful synthesis techniques for them, and to motivate further research into relevant control related issues. One such application area was Formation Flight control.

The key feature of formation flight systems is that they consist of many similar units which are required to follow predetermined trajectories. The goal of control design is to design for disturbance rejection (for physically meaningful disturbance classes), subject to model uncertainty (due to the complexity of fluid-structure interactions, reduced order models will need to be used, leading to model uncertainty), for many interacting units (leading to distributed control).

During the grant period, we completed the construction of an experimental platform at Cornell University that will be used to explore aircraft formation flight control. A schematic representation of the experiment may be found in Figure 4. It consists of four rectangular lifting surfaces, or wings, in a windtunnel with two degrees of freedom: roll about the roll axis, and translation about the pitch axis; the translation about the pitch axis is implemented with a rotary arm, which well approximates translation for small displacements. A fifth lifting surface, a half of one wing, is firmly attached to the side of the wind tunnel and is used to generate a vortical disturbance, which propagates to the downstream wing. The relative spacing of the wings can be adjusted, as can the pitch angle of each wing. Ailerons on each wing, actuated by two micro-servos, provide the only control for each wing. Two optical encoders are used to measure the roll angle and displacement along the pitch axis. The main physical parameters of the system are given in Table 2.

Parameter	Value
Wing airfoil	NACA0015
Wing span	12 inches
Wing chord	3 inches
Effective wing mass along pitch axis	adjustable, > 0.05 kg
Wing roll inertia	adjustable, > 0.0005 kg m^2
Actuator rate limit	11 rad/s
Sensor resolution	0.0015 rad
Wind tunnel cross-section	height 4 feet, width 5 feet
Wind tunnel maximum velocity	30 m/s

Table 2: Physical Parameters

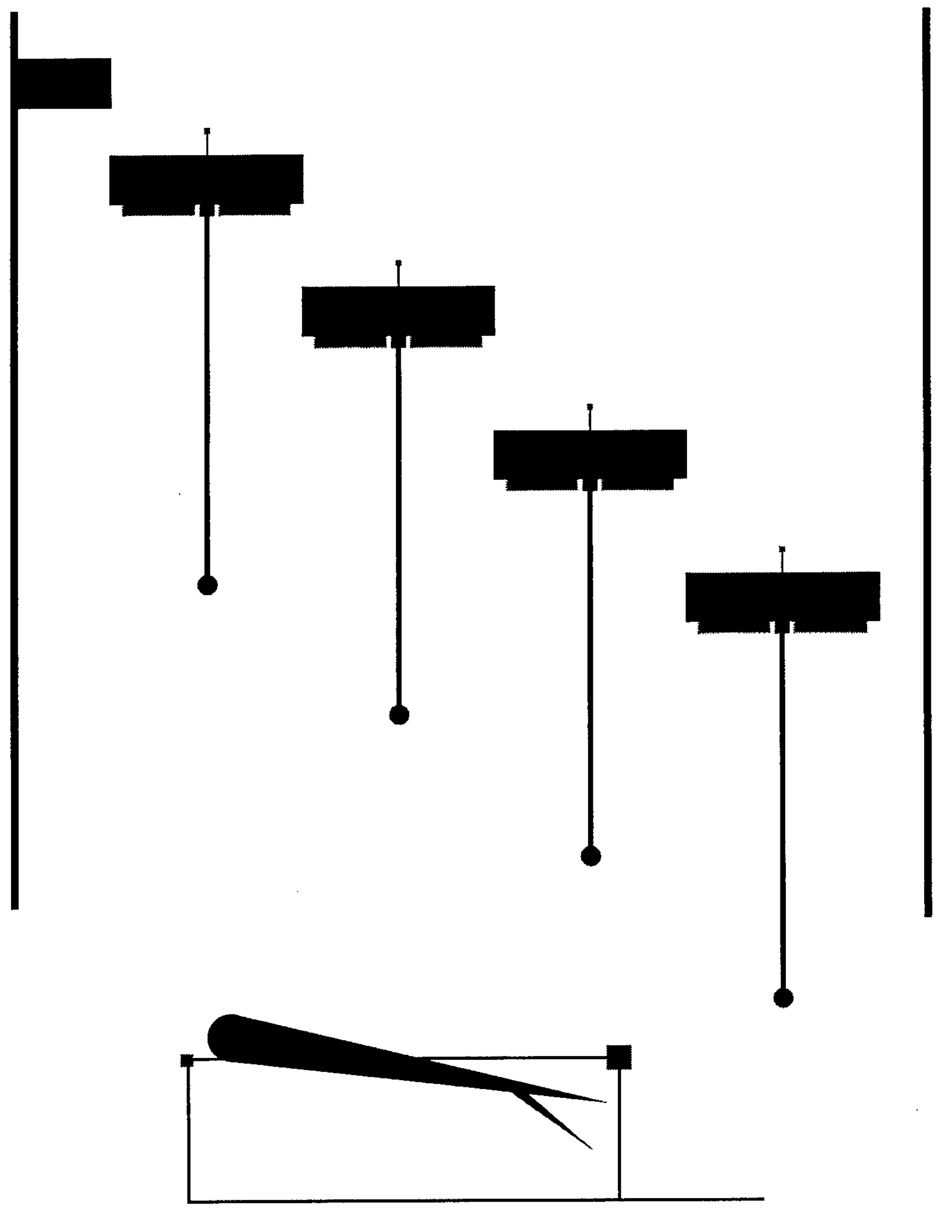

Figure 4: Schematic diagram of experiment. Top diagram, top view; the direction of the flow is from the top of the page to the bottom of the page. The degrees of freedom are roll (the vertical axis of the page), and translation about the pitch axis (the complete wing mount can rotate about an axis coming out of the page, which for small rotations well approximates horizontal translations). Bottom diagram, side view.

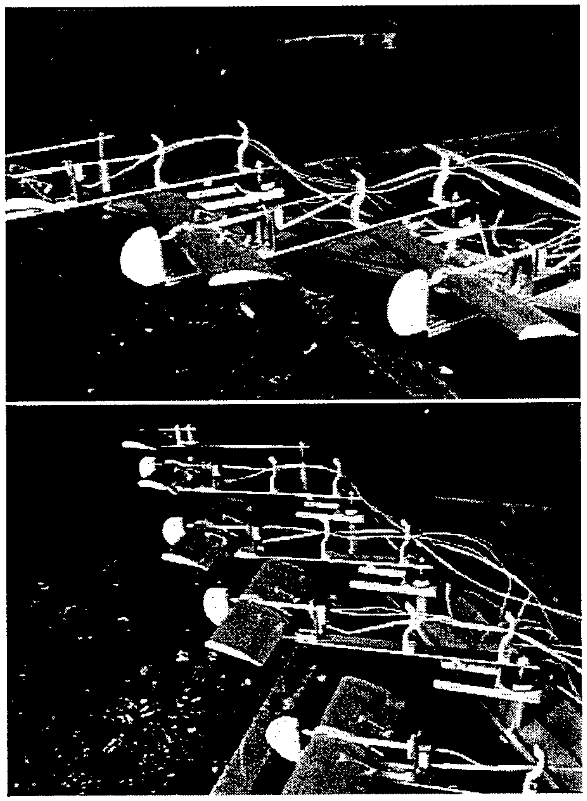

Figure 5: Formation Flight Testbed.

The experiment is instrumented to a Dual Processor Pentium II machine via the DS1103 real time control hardware and software package from dSpace. The dSpace system is being used to implement real time control, while the dual processor workstation is being used for control design and analysis.

4.2 Analytical Models

We have derived simple models which qualitatively capture the dynamics of formation flight systems by capturing the instantaneous lift generated by the wings via vortex sheets in potential flow; see [27] for modeling details. Restricting the analysis to the roll dynamics (no translation), the lift on a downstream wing induced by a moment step on the upstream wing is captured in the left plot of Figure 6, in non-dimensional units; the net induced moment is depicted in the right plot of Figure 6.

This induced moment is the result of the induced velocity associated with the vortex sheet being shed by the upstream wing. The linearized equations of motion for an infinite number

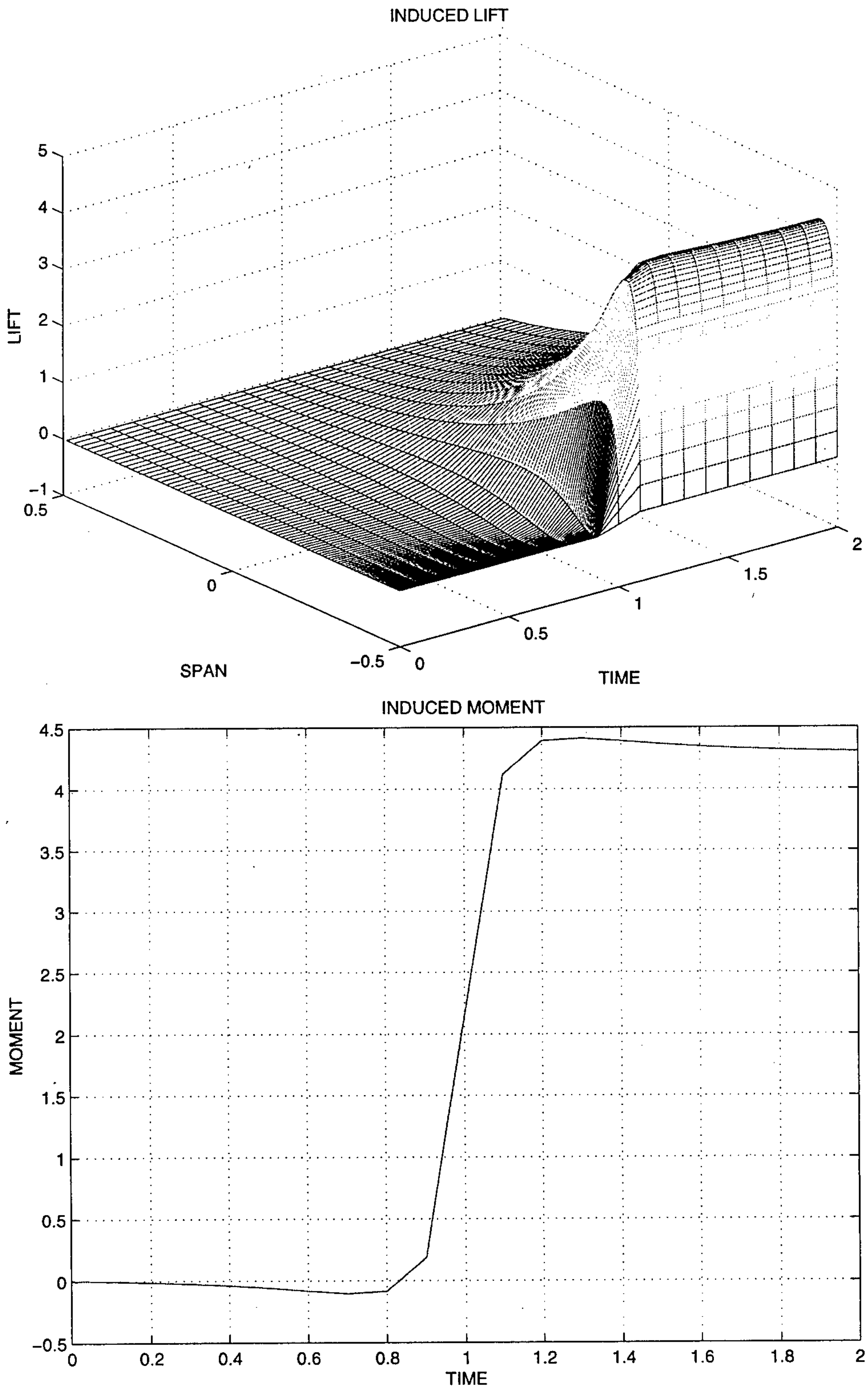

Figure 6: Induced Lift and Moment

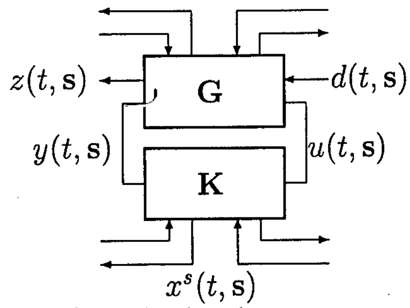

Figure 7: Control architecture. **G** is the local system, coupled to its nearest neighbors. Controller **K** inherits the same architecture.

of wings can be captured by the following operator equation:

$$\mathbf{T}^2\theta = -k\mathbf{T}\theta + \tau + r\mathbf{SD}(1 - p\mathbf{SD})^{-1}\mathbf{T}^2\theta \tag{22}$$

where $\theta = \theta(s,t)$ is the roll angle, independent variable s is the wing number, independent variable t is time, **T** is the temporal differentiation operator

$$(\mathbf{T}\theta)(s,t) := \frac{\partial\theta}{\partial t}(s,t), \tag{23}$$

S is the unit spatial shift operator,

$$(\mathbf{S}\theta)(s,t) := \theta(s-1,t), \tag{24}$$

D is the unit time delay operator,

$$(\mathbf{D}\theta)(s,t) := \theta(s,t-1), \tag{25}$$

$\tau = \tau(s,t)$ is the torque applied to the wing, and k, r, and p are constants derived from the physical parameters of the system.

The simple model above is valid for an infinite number of wings; given that the coupling is only from upstream wings to downstream wings, however, controllers designed for the infinite wing case will guarantee stability and performance for systems consisting of only a finite number of wings. Controllers were designed using the techniques outlined in [16] and [17] for the models described above. A frame of a simulation may be found in Figure 8, corresponding to time = 45 units. The left most wing is the lead wing, the right most wing is the trailing wing. Random noise torques were applied to each of the wings throughout the simulation.

One of the attractive features of using the multidimensional optimization techniques in [16] and [17] to these classes of problems is that the resulting control architecture mimics that of the plant. This is depicted in Figure 7. In particular, the computation is decentralized, with the controllers **K** communicating with their nearest neighbors.

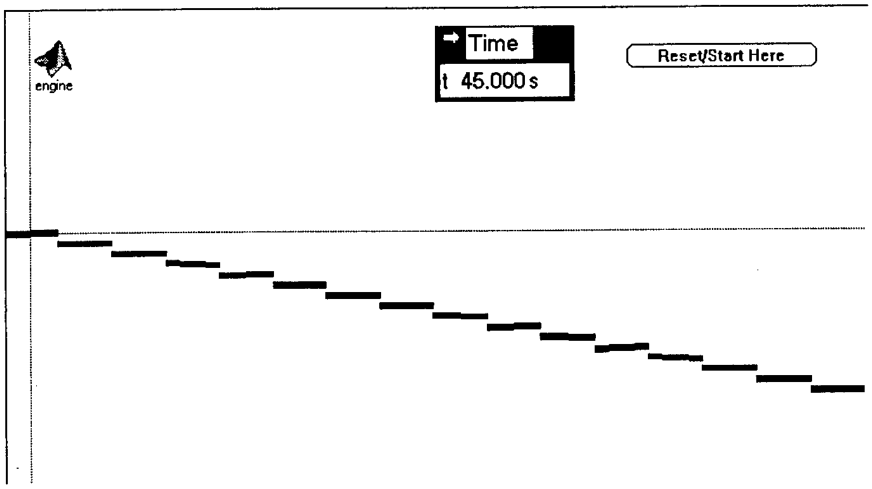

Figure 8: Closed loop system, multidimensional system optimization. The view is essentially a rear view, with an elevation of 20 degrees to separate the wings (the roll axes of the wings are essentially into the page). The left-most wing is the lead wing.

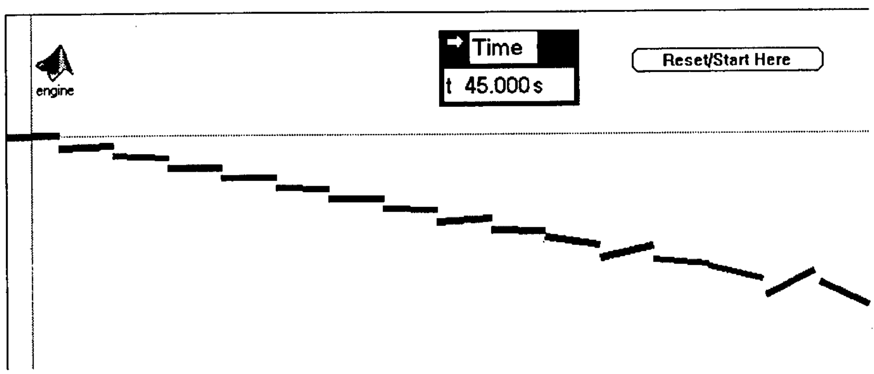

Figure 9: Closed loop system, decentralized $\mathcal{H}_\infty$ optimization.

Using the same weights and performance objectives, decentralized controllers were designed using $\mathcal{H}_\infty$ optimization by treating the fluid coupling as a disturbance. A frame of a simulation may be found in Figure 9, corresponding to time = 45 units. As can be seen, the resulting closed loop system displays string instabilities.

4.3 Future Work

As a follow up to this grant, we are in the process of constructing models which incorporate the translational, actuator, and sensor dynamics, and thus provide a first principles model of the complete experimental system about the natural equilibrium point. Robust linear

identification and model validation techniques are being employed to incorporate experimental data in the resulting model, and to extend the model to include system uncertainty. The various system parameters, such as pitch angle, mass of the wing, velocity of the flow in the wind tunnel, etc., are being varied in order to control the difficulty of controlling the system with purely decentralized schemes.

References

[1] H. Raza and P. Ioannou. Vehicle following control design for automated highway systems. *IEEE Control Systems*, 16(6):43–60, 1996.

[2] Lissaman and Shollenberger. Formation flight of birds. *Science*, 168:1003–1005, 1970.

[3] J.D. Wolfe, D. F. Chichka, and J. L. Speyer. Decentralized controllers for unmmaned aerial vehicle formation flight. *American Institute of Aeronautics and Astronautics*, 96-3833, 1996.

[4] D. F. Chichka and J. L. Speyer. Solar-powered, formation-enhanced aerial vehicle system for sustained endurance. In *Proc. American Control Conference*, pages 684–688, 1998.

[5] G. B. Shaw, D. W. Miller, and D. E. Hastings. *The Generalized Information Network Analysis Methodology for Distributed Satellite Systems*. PhD thesis, Massachusetts Institute of Technology, 1998.

[6] V. Kapila, A. G. Sparks, J. Buffington, and Q. Yan. Spacecraft formation fying: Dynamics and control. In *Proc. American Control Conference*, pages 4137–4141, 1999.

[7] A. D. Robertson, G. Inalhan, and J. P. How. Formation control strategies for a separated spacecraft interferometer. In *Proc. American Control Conference*, pages 4142–4147, 1999.

[8] M. Mesbahi and F. Y. Hadaegh. Graphs, matrix inequalities, and switching for the formation flying control of multiple spacecraft. In *Proc. American Control Conference*, pages 4148–4152, 1999.

[9] D. L. Laughlin, M. Morari, and R. D. Braatz. Robust performance of cross-directional control systems for web processes. *Automatica*, 29(6):1395–1410, 1993.

[10] G.E. Stewart. *Two dimensional loop shaping controller design for paper machine cross-directional processes*. PhD thesis, University of British Columbia, 2000.

[11] G.E. Stewart, D. M. Gorinevsky, and G.A. Dumont. Spatial loopshaping: A case study on cross-directional profile control. In *Proc. American Control Conference*, pages 3098–3103, 1999.

[12] M. E. Taylor. *Partial Differential Equations I: Basic Theory*. Springer Verlag, 1996.

[13] H. T. Banks, R. C. Smith, and Y. Wang. *Smart Material Structures Modeling, Estimation, and Control*. John Wiley and Sons, 1996.

[14] R. D'Andrea and G. E. Dullerud. Distributed control of spatially interconnected systems. *IEEE Transactions on Automatic Control.* Submitted for publication.

[15] R. D'Andrea. Linear matrix inequalities, multidimensional system optimization, and control of spatially distributed systems: An example. In *Proc. American Control Conference*, pages 2713–2717, 1999.

[16] R. D'Andrea. A linear matrix inequality approach to decentralized control of distributed parameter systems. In *Proc. American Control Conference*, pages 1350–1354, 1998.

[17] R. D'Andrea, G. Dullerud, and S. Lall. Convex ℓ_2 synthesis for multidimensional systems. In *Proc. IEEE Conference on Decision and Control*, pages 1883–1888, 1998.

[18] B. Bamieh. The structure of optimal controllers of spatially invariant distributed parameter systems. In *Proc. IEEE Conference on Decision and Control*, pages 1056–1061, 1997.

[19] B. Bamieh, F. Paganini, and M. Dahleh. Optimal control of distributed actuator and sensor arrays. In *SPIE conference on Smart Structures*, pages 467–478, 1998.

[20] M. L. El-Sayed and P. S. Krishnaprasad. Homogeneous interconnected systems: An example. *IEEE Transactions on Automatic Control*, 26(4):894–901, 1981.

[21] R. D'Andrea and F. Paganini. Interconnection of uncertain behavioral systems for robust control. In *Proc. IEEE Conference on Decision and Control*, pages 3642–3647, 1993.

[22] J. C. Willems. Paradigms and puzzles in the theory of dynamical systems. *IEEE Transactions on Automatic Control*, 36:259–294, 1991.

[23] R. D'Andrea, F. Paganini, and J. C. Doyle. Uncertain behavior. In *Proc. IEEE Conference on Decision and Control*, pages 3891–3896, 1993.

[24] R. D'Andrea and S. Khatri. Kalman decomposition of linear fractional transformation representations and minimality. In *Proc. American Control Conference*, pages 3557–3561, 1997.

[25] C. L. Beck and R. D'Andrea. Computational study and comparisons of LFT reducibility methods. In *Proc. American Control Conference*, pages 1013–1017, 1998.

[26] G. E. Dullerud, R. D'Andrea, and S. Lall. Control of spatially varying distributed systems. In *Proc. IEEE Conference on Decision and Control*, pages 1889–1893, 1998.

[27] J. D. Anderson. *Fundamentals of Aerodynamics*. McGraw Hill, 1991.